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$$y=\frac{(a\sqrt{3}+b)(a\sin\phi+b\cos\phi)}{2(a^2\sin^2\phi+b^2\cos^2\phi)^{\frac{1}{2}}} \text{ and } x=\frac{(a+b\surd 3)(b\cos\phi-a\sin\phi)}{2(a^2\sin^2\phi+b^2\cos^2\phi)^{\frac{1}{2}}}.$$

Squaring and adding these two equations we have

$$\frac{y^2}{(b+a\sqrt{3})^2}+\frac{x^2}{(a+b\sqrt{3})^2}=\frac{1}{2}$$ or the equation of the required locus.

Also solved in a very excellent manner by *Professors ALFRED HUME* and *G. B. M. ZERR.*

13. Proposed by HENRY HEATON, M. S., Atlantic, Iowa.

Tiough two given points to pass four spherical surfaces tangent to two given spheres.

I. Solution by the PROPOSER.

On lines through the external center of similitude and the given points, two other points may be found through which the spherical surface must pass if the given spheres are tangent to it either both externally or both internally.

These points are found in a manner exactly similar to that in which the point U is found in the solution to problem 14. A circle will pass through the four points, which is evidently the circle in which the plane of the two lines through the points cuts the required spherical surface or surfaces.

If a plane be passed through the center of either of the given spheres and that of either of the required spherical surfaces it will pass through the the point of tangency and the section will be great circles of the two spheres tangent to each other. It will cut the circle through the four points in two other points.

Hence if we pass a plane through the center of either of the given spheres and through the center of the circle through the points and perpendicular to its plane, the section will be a great circle of the given sphere and two points of the circle. Circles through these two points tangent to the circle thus found, will be great circles of two of the required spherical surfaces. The spherical surfaces through these great circles, will be those to which the two given spheres are tangent either both externally or both internally.

The other two spherical surfaces may be found in a manner exactly similar by taking points on lines through the internal center of similitude.

II Solution by H. C. WHITAKER, B. S., M E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Let the equation of the required sphere be

$$(x-l)^2+(y-m)^2+(z-n)^2-R^2=0.$$

Then we have using the first condition

$$(a_1-l)^2+(b_1-m)^2+(c_1-n)^2-(R\pm r_1)^2=0.$$

And also by using the second condition

$$(a_2-l)^2+(b_2-m)^2+(c_2-n)^2-(R\pm r_2)^2=0.$$

And also by using the third condition

$$(x_1-l)^2+(y_2-m)^2+(z_2-n)^2-R^2=0.$$

And also by using the fourth condition

$$(x_2-l)^2+(y_2\quad m)^2+(z_2-n)^2-R^2=0.$$

The required equation is found by eliminating l, m, n and R giving the equation,

$$\left|\begin{array}{lllll} x^2+y^2+z^2 & x & y & z & 1 \\ a_1{}^2+b_1{}^2+c_1{}^2 & a_1 & b_1 & c_1 & (1\pm r_1)^2 \\ a_2{}^2+b_2{}^2+c_2{}^2 & a_2 & b_2 & c_2 & (1\pm r_2)^2 \\ x_1{}^2+y_1{}^2+z_1{}^2 & x_1 & y_1 & z_1 & 1 \\ x_2{}^2+y_2{}^2+z_2{}^2 & x^2 & y^2 & z^2 & 1. \end{array}\right\} =0.$$

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CALCULUS

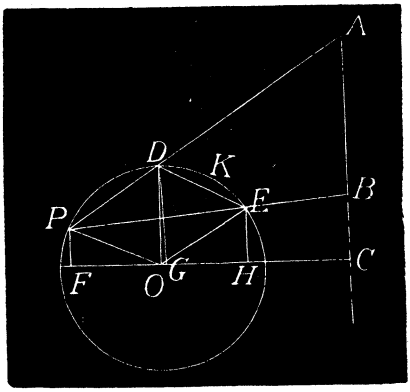
Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

10. **Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa.**

Prove or disprove the following theorem: If O be any circle, and AB any straight line either within or without the circumference, and if a perpendicular be dropped from O upon AB and prolonged backward to meet the circumference in P, *then* will the angle whose vertex lies at P and whose sides pass through A and B, cut a portion CPD from the circle which shall be greater than that cut by any other angle whose vertex lies on the circumference and whose sides pass through A and B.

Solution by G. B. M. ZERR, Principal of High School, Staunton, Virginia.



Let O be the centre of the given circle A, B the given points in the line ABC, OC the perpendicular from O on ABC, P any point in the circumference of the circle, D the point where AP cuts the circumference, E the point where PB cuts the circumference.

Let (x_1,y_1) be the coordinates of P, and $(a,b,)$ and (a,c) the coordinates of A, and B; O the origin then $x^2+y^2=r^2$ is the equation to the circle, also $x_1{}^2+y_1{}^2=r^2$.

Equation to PA is $(a-x_1)y+(y_1-b)x=ay_1-bx_1\ldots.(2)$.

Equation to PB is $(a-x_1)y+(y_1-c)x=ay_1-cx_1\ldots.(3)$.

Eliminating x between $x^2+y^2=r^2$ and (2) we get $y^2+\dfrac{2y(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y_1)^2}=\dfrac{r^2(b-y_1)^2-(bx_1-ay_1)^2}{(a-x_1)^2+(b-y_1)^2}$.

$$\therefore\ y+\frac{(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y)^2}=\pm\frac{(b-y_1)\sqrt{r^2(b-y_2)^2+r^2(a-x_1)^2-(bx_1-ay_1)^2}}{(a-x_1)^2+(b-y_1)^2}$$